Coordinate Geometry

Definition. *Any line in the plane can be described by an equation of the form*

$$ax + by = c$$

for some $a, b, c \in \mathbb{R}$.

If the line is not vertical, then it can be described by an equation of the form

$$y = mx + c$$

for some $m, c \in \mathbb{R}$ where m is the gradient of the line and c is the y-intercept. Also, if we know the line goes through a point (x_1, y_1) and has gradient m, then it can be described by an equation of the form

$$y - y_1 = m(x - x_1).$$

Theorem. *Suppose we have two lines $y = m_1x + c_1$ and $y = m_2x + c_2$.*

(i) The lines are parallel iff $m_1 = m_2$.
(ii) The lines are perpendicular iff $m_1m_2 = -1$.

Suppose we have two lines in the plane, then there are three possible configurations:

(i) the lines intersect in a unique point,
(ii) the lines are parallel, but disjoint, or
(iii) the lines are coincident.

When the lines intersect in a unique point we can solve the simultaneous equations to identify the coordinates of the point of intersection. This is achieved by substitution or elimination.

Theorem (Pythagorean Theorem). *Suppose we have a triangle with side lengths a, b, c, and associated angles A, B, C. If C is a right angle, then*

$$a^2 + b^2 = c^2.$$

Theorem (Converse to the Pythagorean Theorem). *Suppose we have a triangle with side lengths a, b, c, and associated angles A, B, C. If*

$$a^2 + b^2 = c^2,$$

then C is a right angle.

There are various generalizations of the Pythagorean Theorem to higher dimensions, non-right angles etc. but we will not discuss these here.

Remark. *The most important consequence of the Pythagorean Theorem is that it allows us to calculate the distance between two points. The distance between (x_1, y_1) and (x_2, y_2) is given by*

$$\textit{distance} = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}.$$