Natural Numbers $\mathbb{N} = \{1, 2, 3, ...\}$.

Integers $\mathbb{Z} = \{... -2, -1, 0, 1, 2, ...\}$.

Rational Numbers $\mathbb{Q} = \{\text{all fractions}\}$.

Real Numbers $\mathbb{R} = \left\{\text{all fractions and numbers like } \sqrt{2}, \sqrt{15}, \pi, ...\right\}$.

Note that we have the following inclusions:

$$\mathbb{N} \subset \mathbb{Z} \subset \mathbb{Q} \subset \mathbb{R}\,.$$

Systems for expressing numbers using a select list of digits are known as **base number systems**. Specifically, when we express numbers as sums of powers of a fixed integer b, we say that it is written in base b. In practice this allows us to express numbers using only the first b integers $0, 1, 2, 3, ..., b-1$, we say that we are writing numbers in base b. For example, the **decimal number system** is base 10, and the **binary number system** is base 2. A less common example is the **hexadecimal number system** which uses the sixteen digits: $0, 1, 2, 3, 4, 5, 6, 7, 8, 9$,A,B,C,D,E,F. In principle, we could use any positive integer as the base of our number system.

We typically represent numbers using the decimal number system. A number whose decimal representation ends after a finite number of digits is said to have a **terminating decimal expansion**. A number whose decimal representation consists of an endlessly repeating sequence of digits is said to have a **repeating decimal expansion**.

Every number has a decimal expansion, but not every number can be represented as a fraction. Numbers which can be written as a fraction are known as **rational numbers** and numbers which cannot be written as a fraction are known as **irrational numbers**.

- $\mathbb{R}$ → Rational Numbers
  - → Numbers with terminating decimal expansions (these are precisely those fractions which, when fully simplified, only have powers of 2 and 5 in their denominators). i.e. $\frac{1}{2}, \frac{3}{25}, \frac{7}{50}....$
  - → Numbers with endless, repeating decimal expansions (these are precisely those fractions which, when fully simplified, have a prime other than 2 or 5 in their denominators). i.e. $\frac{1}{7}, \frac{15}{29}, -\frac{2}{11}....$
- $\mathbb{R}$ → Irrational Numbers
  - → Numbers with endless, never-repeating decimal expansions (these are precisely those numbers which cannot be expressed as fractions). i.e. $\sqrt{2} = 1.41421356237....$