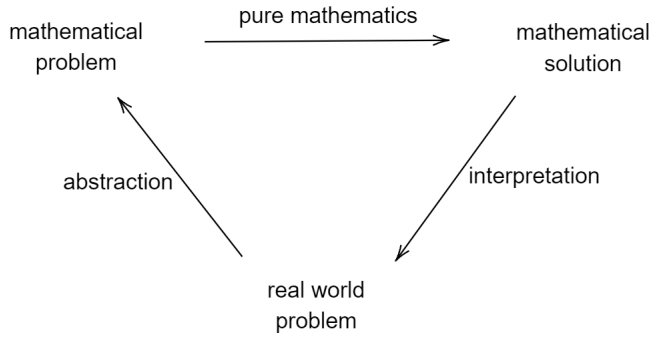
Mathematical Culture

Mathematics is often partitioned into the sub-disciplines of **pure mathematics** and **applied mathematics**. Pure mathematics is concerned with the study of abstract structures and patterns. It is the study of mathematics for its own sake. Applied mathematics is summarized in the diagram below. Given a physical situation, a mathematical model is constructed by making simplifications and abstracting away unnecessary details. The real world problem is then formulated within the model as a well-defined mathematical problem, and solved using mathematical techniques. Once a solution is obtained it must be interpreted in light of the original physical context. This last step requires a careful analysis of the assumptions underpinning the model to determine the validity of the solution.



Often the theory developed through the investigation of questions in pure mathematics finds surprising and useful applications to problems of applied mathematics.

The highest honour a person can receive in mathematics is a **Fields Medal**. It is equivalent in prestige to the Nobel prizes in the sciences (there is no Nobel for mathematics), and is a very big deal. It is awarded to up to four mathematicians, once every four years. The medal was first awarded in 1936, and it is named after the Canadian mathematician who established it. You can only win the medal if you are under 40 years old. This was intended to encourage young mathematicians to tackle the hardest open problems.

The **International Congress of Mathematicians** (ICM) is a major conference, held every four years, bringing together many of the most prominent mathematicians. In particular, the winners of the Fields medal are announced at the ICM. In 1900 the conference was held in Paris, and it was here that David Hilbert, the best mathematician at the time, presented a list of 23 open problems which he deemed to be the most important problems for mathematicians to solve over the ensuing years. This list set the research agenda of the worldwide mathematics community for much of the 20th century. Almost every problem on the original list has since been solved.

In 1998 the Clay Mathematics Institute was founded by wealthy American businessman, Landon Clay. The organization exists to promote mathematics research. In 2000 the Clay Mathematics Institute organized for an updated list of problems to be devised, analogous to Hilbert's list in 1900. This new list contained seven problems which were regarded by the mathematics community as being the most difficult and important problems to solve in the coming century. The resolution of any one of these problems would represent a major scientific breakthrough. They are called the **Millennium Problems** and each carries a $1 million prize.

Only one of the problems on Hilbert's initial list remains as a Millennium Problem. It is the **Riemann Hypothesis** - the most celebrated and highly coveted mathematics problem in the world. For over 150 years many of the best mathematicians in the world have tried to prove or disprove this hypothesis, and it still remains unsolved. The problem has to do with understanding the distribution of the prime numbers amongst the integers. Anybody who resolves the Riemann Hypothesis will be crowned with glory.

The other six problems on the list are: the **Navier-Stokes equations**, **P versus NP**, the **Yang-Mills** problem, the **Hodge Conjecture**, the **Birch and Swinnerton-Dyer Conjecture**, and the **Poincare Conjecture**. They all remain unsolved except for the Poincare Conjecture. The Poincare Conjecture has a very strange story behind it. In 2002 and 2003 a reclusive Russian mathematician named Grigori Perelman posted three articles on the internet which claimed to give a proof of the Poincare Conjecture. It took several years

for others to verify that what he had written was indeed correct. Subsequently, he was offered the Fields medal and the $1 million prize - both of which he refused. He has since withdrawn from the mathematical community.

There are prestigious mathematical accolades apart from the Fields Medal, such as the **Abel Prize** and the **Breakthrough Prize in Mathematics**, amongst others.

The most prestigious mathematics competitions in the world are the **International Mathematical Olympiad**, for high school students (each country sends a team of six competitors), and the **Putnam Mathematical Competition**, for undergraduate students (students compete individually). They are both *notoriously* difficult. In addition, most developed countries have a range of other mathematics competitions for high school students, of varying degrees of difficulty.

The following institutions are renowned for their mathematics departments (in no particular order): **Princeton**, **MIT**, **Harvard**, **Stanford** and **Berkeley** (in the US), **Cambridge** and **Oxford** (in the UK) and **Bonn** (in Germany). Such lists are inevitably subjective, and numerous other institutions could be added.

There are many interesting characters in the history of mathematics, including **Newton**, **Fermat**, **Euler**, **Gauss**, **Galois**, **Ramanujan**, **Cantor**, **Turing**, **Hilbert**, **Noether**, **Gödel**, amongst many others.

Mathematical research (as with almost all research) is published in academic journals. There are thousands of journals dedicated to mathematics, and they vary widely in quality and prestige. Typically, when a mathematician writes a paper they will submit it for publication to a journal of a standard which they deem to be commensurate with the quality and level of impact of the paper itself. The journal will then organize for the paper to be reviewed by an expert, who will recommend that the paper be accepted for publication, or rejected. The reviewing process often takes a long time (sometimes years). For example, arguably the most prestigious journal in pure mathematics is **Annals of Mathematics**. It is extremely difficult to get a paper published in this journal. The vast majority of mathematicians go their entire career without producing anything deemed worthy to be included in Annals. You can browse through the titles and abstracts of papers from the different issues on the website. Do not be intimidated if you barely understand any of the content - academic papers are generally written in a very terse style, with the intended audience being specialists in specific branches of mathematics who have studied for many years. Because the process of getting a paper published takes so long, and because papers published in journals are only accessible to people who belong to institutions that pay the journal's subscription fees, mathematicians usually simultaneously submit their new papers to an online repository called the **arXiv** (pronounced "archive"). This provides a central location for storing research which is freely accessible to all. The reason people bother submitting to journals is because there is no reviewing mechanism on the arXiv, and there is no guarantee that what is contained there is correct. Nonetheless, it is very widely used, not just by mathematicians, but also those in neighbouring disciplines.

The standard document typesetting program for research in mathematics is known as **Latex**. Essentially every article on modern mathematics (and many articles in neighbouring disciplines) is written using this software. With Latex you use a text editor to type your content, prefaced by a brief technical preamble, and then compile this code to render the document as a pdf. In contrast to, say, Microsoft Word, where what-you-see-is-what-you-get, with Latex the document appearance is determined by typing commands. The whole idea is that Latex allows the author to focus on the content of their document, whilst it automates the handling of the formatting, like the spacing of equations etc. Moreover, it is highly customizable and gives the author a great deal of control over a document's presentation.

Quanta magazine is an online publication that is arguably the best place to read about cutting edge developments in mathematics if you are not a specialist. The articles are generally accessible whilst still being accurate to the subject at hand.