Set Theory

Definition. *A* ***set*** *is a collection of objects. The objects in a set are called* ***elements*** *or members of a set. We denote sets using curly brackets. For example,*

$$\{1, 2, 3\}$$

is the set containing the numbers $1, 2$ *and* 3*. For a set* A *we write* $x \in A$ *if* x *is an element of* A*, and* $x \notin A$ *if* x *is not an element of* A*.*

Definition. *Suppose we have two sets* A *and* B*.*

- *(i) If every element of* A *is an element of* B *we say that* A *is a* ***subset*** *of* B*, and write* $A \subset B$*.*
- *(ii) The* ***intersection*** *of* A *and* B *is a new set written* $A \cap B$ *which contains precisely those elements which are in both* A *and* B*.*
- *(iii) The* ***union*** *of* A *and* B *is a new set written* $A \cup B$ *which contains precisely whose elements which either in* A *or in* B*, or in both.*

Example. *Suppose* $A = \{1, 2, 3\}$*, then the set containing all the subsets of* A *is*

$$\Big\{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\}\Big\}$$

Definition. *Suppose we have two sets* A *and* B*. If* A *and* B *have no elements in common, then we say that they are* ***disjoint****, and write* $A \cap B = \emptyset$*. The symbol* $\emptyset$ *denotes the* ***empty set****, that is the set containing no elements.*

Definition. *Suppose we have two sets* A *and* B*. We use the notation* $A \backslash B$ *to denote the set of all elements in* A *that are not in* B*. Similarly,* $B \backslash A$ *denotes the set of all elements in* B *that are not in* A*.*

Definition. *We define the* ***cardinality*** *of a set* A *to be the number of elements the set contains. The cardinality of a set may be finite or infinite.*

The number systems are very important examples of sets that contain infinitely many elements. The number systems are denoted by:

- $\mathbb{N}$ for the natural numbers,
- $\mathbb{Z}$ for the integers,
- $\mathbb{Q}$ for the rational numbers,
- $\mathbb{R}$ for the real numbers.

Definition. *Sometimes we will be working exclusively within a fixed set. In such situations we call this set our* ***universal set****. Suppose that* A *is a subset of our universal set* Ω*. We define the* ***complement*** *of* A *(within* Ω*) to be the set of all elements of* Ω *that are not elements of* A*. Often the complement of* A *is written as* A'*.*

Examples of Sets.

(i) $\Big\{x \in \mathbb{R} : 0 < x < 3\Big\}$

(ii) $\Big\{x \in \mathbb{Z} : x \leq -5\Big\}$

(iii) $\Big\{x \in \mathbb{R} : 0 < x < 3\Big\} \cap \Big\{x \in \mathbb{Z}\Big\}$

(iv) $\Big\{x \in \mathbb{Q} : x > 1\Big\}$

(v) $\Big\{2n + 1 : n \in \mathbb{N}\Big\}$

(vi) $\Big\{x \in \mathbb{R} : 0 < x < 10\Big\} \cup \Big\{17\Big\}$

We often denote **intervals** of real numbers using the following notation. The key point is that square brackets include the endpoint, but rounded brackets don't include the endpoint. Suppose $c, d \in \mathbb{R}$ such that $c < d$.

(i) $(c, d) = \Big\{x \in \mathbb{R} : c < x < d\Big\}$

(ii) $(c, d] = \Big\{x \in \mathbb{R} : c < x \leq d\Big\}$

(iii) $(c, \infty) = \Big\{x \in \mathbb{R} : c < x < \infty\Big\}$

(iv) $(-\infty, d) = \Big\{x \in \mathbb{R} : -\infty < x < d\Big\}$

(v) $[c, d] = \Big\{x \in \mathbb{R} : c \leq x \leq d\Big\}$

(vi) $[c, d) = \Big\{x \in \mathbb{R} : c \leq x < d\Big\}$

(vii) $[c, \infty) = \Big\{x \in \mathbb{R} : c \leq x < \infty\Big\}$

(viii) $(-\infty, d] = \Big\{x \in \mathbb{R} : -\infty < x \leq d\Big\}$

We also use the notation $\mathbb{R}^2$ to refer to the set of all ordered pairs of real numbers, that is, the Cartesian plane. Similarly, we use the notation to $\mathbb{R}^3$ to refer to the set of all ordered triplets of real numbers, that is, 3-dimensional space.